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intersection of two pairs of coincident planes, i. e., it is a quadruple line, and the points where it pierces φ are quadruple points common to the surfaces (1) and (2). If we subtract (2) from (1) the resulting equation will be that of some surface passing through the intersection of (1) and (2). Thus we have

$$(\lambda_1-\lambda_2)V^2+(\mu_1-\mu_2)W^2=0,$$

or

$$V\sqrt{(\lambda_1-\lambda_2)}=\pm W\sqrt{(\mu_3-\mu_1)}. \qquad (3)$$

Equation (3) represents two real or imaginary planes, according to the signs and relative values of λ_1, λ_2 and μ_1, μ_2. But these planes contain the intersection of (1) and (2), which consists therefore of two plane curves, real or imaginary. Q. E. D.

The particular case of this proposition when $\varphi(U_1, U_2 \text{ etc.}) = U_1U_2$, and U_1 and U_2 are of the 1st degree, or, in general, when φ is of the 2nd degree, proves directly the property at which I arrived indirectly in an article published in the ANALYST for July, 1876; viz., Two quadrics will intersect each other in plane curves when they have common tangent planes at the extremities of a common chord.

We may also write the reciprocal proposition to this as follows:

If two surfaces of the same class are tangent to each other at the points of contact of all planes that can be drawn tangent to them through a given right line, then they will have two common circumscribing cones; i. e., every common tangent plane of the two surfaces will pass through one or the other of two fixed points on the given line.

SOLUTION OF A PROBLEM.

BY GEORGE EASTWOOD, SAXONVILLE, MASS.

Problem.—Now that there are three tickets before the people, from which to elect our next President, let it be supposed that the votes will be divided between the candidates, on the assumption that the voters will attribute to the three tickets every degree of merit comprised within the limits 0 and 100. Supposing the voters to find in each ticket some—*more* or *less*—of every degree of merit contained in the other two, it is required to assign the mean value of *all* the the merits of the three tickets.

Solution.—That the question may have the utmost generality, let us suppose the number of tickets to be n, and the limits of their merits to be between 0 and a. Suppose β_1, β_2 . . . . . β_r represent the respective merits of the n tickets, β_1 being the merit attributed to the ticket which the voter deems the most worthy, β_2 the next &c., β_r being the merit ascribed to the rth ticket in point of merit.

First, assume that β_1 shall have any positive value comprised within the limits 0 and a, and that the mean value of all such values of β_1 is required. Let $v =$ the number of values taken and $\Sigma\beta_1$ = their sum; then the req'd mean of all the values is

$$M = \Sigma\beta_1 \div v.$$

If we suppose the values of β_1 to increase by equal portions δ, and that they become $0, \delta, 2\delta \ldots\ldots a$, then

$$v = \Sigma\delta \div \delta, \text{ and } M = \delta\Sigma\beta_1 \div \Sigma\delta.$$

Now because β_1 is supposed to receive all values between 0 and a, the portion δ may be taken indefinitely small, say $= d\beta_1$, and the sums Σ may be replaced by integrals.

$$\therefore M = \int_0^a \beta_1 d\beta_1 \div \int_0^a d\beta_1 = \frac{\frac{1}{2}a^2}{a} = \tfrac{1}{2}a.$$

Hence the mean value of all these values is equal to the mean of the two extremes.

Similarly, suppose β_1, β_2 to be the merits of two tickets according to the opinion of any independent voter, β_1 being the highest estimate he places on the most worthy ticket; so that β_1 is greater than β_2. In this case β_2 may have any values between 0 and β_1, and β_1 any value between $\beta_2 = 0$ and $\beta_2 = \beta_1$. The sum of all the values of β_1 contained between $\beta_2 = 0$ and $\beta_2 = \beta_1$ is equal to

$$\beta_1 \int_0^{\beta_1} d\beta_2 \div d\beta_2,$$

and the sum of all the values of β_1 is equal to

$$\int_0^a \left(\beta_1 d\beta_1 \int_0^{\beta_1} d\beta_2\right) \div d\beta_1 d\beta_2.$$

The number of these values is

$$\int_0^a d\beta_1 \int_0^{\beta_1} d\beta_2 \div d\beta_1 d\beta_2,$$

therefore the mean value of β_1 is

$$\int_0^a \int_0^{\beta_1} \beta_1 d\beta_1 d\beta_2 \div \int_0^a \int_0^{\beta_1} d\beta_1 d\beta_2.$$

Thirdly, if there be n tickets and β_r be the merit attributed to the rth ticket, the mean value of β_1 will be

$$\int_0^a \int_0^{\beta_1} \int_0^{\beta_2} \cdots \int_0^{\beta_{n-1}} \beta_r d\beta_1 d\beta_2 \ldots d\beta_n \div \int_0^a \int_0^{\beta_1} \int_0^{\beta_2} \cdots \int_0^{\beta_{n-1}} d\beta_1 d\beta_2 \ldots d\beta_n$$

$$= \frac{1}{2} \cdot \frac{1}{3} \cdots \frac{1}{n-r} \cdot \frac{1}{1-r+2} \cdot \frac{1}{1-r+3} \cdots \frac{1}{n+1} a^{n+1} \div \frac{1}{2} \cdot \frac{1}{3} \cdot \ \frac{1}{n} a^n = \frac{n-r+1}{n+1} a.$$

If n be an odd No. $\beta_{\frac{1}{2}(n+1)}$ will be the middle term and its mean value will be $\frac{1}{2}a$. If n be an even No. $\beta_{\frac{1}{2}n}$ and $\beta_{\frac{1}{2}(n+1)}$ are middle terms; their mean values are $\frac{1}{2}a+\frac{\frac{1}{2}a}{n+1}$ and $\frac{1}{2}a-\frac{\frac{1}{2}a}{n+1}$, and the mean of these values is $\frac{1}{2}a$. The sum of n values is $\frac{1}{2}an$, and hence the mean of all the values is $\frac{1}{2}a$.

Suppose now we take the values of the three tickets, as estimated per question by the independent voters, and that these values are as the Nos. 1, 2, 3. Then, substituting n by 3, a by 100, and r severally by 1, 2, 3, we shall have for the mean value of the most worthy ticket, $\frac{3}{4}\times 100 = 75$,
for the mean value of the next best, $\frac{1}{2}\times 100 = 50$,
for the mean value of the third " , $\frac{1}{4}\times 100 = 25$;
while the *mean* of all the values will be $\frac{1}{3}\times\frac{3}{2}\times 100 = 50$.

SOLUTION OF A PROBLEM.

BY WILLIAM HOOVER, SUPERINTENDENT OF SCHOOLS, WAPAKONETA, O.

Problem.—A triangle is formed by joining the centers of the escribed circles of a triangle; the same is done for the new triangle, and so on *ad infinitum.* Show that the final triangle is equilateral.

Solution.—If A be one of the angles of the given triangle, and A_1, A_2, A_3, &c., the successive corresponding angles, we have

$A_1 = (\frac{1}{2})(\pi - A)$, $A_2 = (\frac{1}{2})^2(\pi + A)$, $A_3 = (\frac{1}{2})^3(3\pi - A)$, $A_4 = (\frac{1}{2})^4(5\pi + A)$.

The fraction for the nth angle is $(\frac{1}{2})^n$, the coefficient of π is the general term of the recurring series 1, 1, 3, 5, &c., $\frac{1}{3}[2^n + (-1)^{n-1}]$, and of A, it is $(-1)^n$. Hence

$$A_n = \frac{1}{2^n}\left[\tfrac{1}{3}\pi[2^n+(-1)^{n-1}]+(-1)^n A\right].$$

When n is infinite $A_n = \frac{1}{3}\pi$, and the triangle is equilateral.

SOLUTION OF PROB. 282 WITH CONDITION (3) CHANGED.

BY CHAS. H. KUMMELL, U. S. LAKE SURVEY.

The solution of this problem (see pp. 25, 26) is in accordance with the given conditions. However it cannot be admitted, as assumed in condition (3), that it is equally probable that only one loom should be out of order as